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Two-body Mechanisms in Pion Photoproduction on the Trinucleon

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Abstract

A breakdown of the Impulse Approximation is studied in pion photoproduction on ^{3}He at high momentum transfers. The usual DWIA formalism with Faddeev wave functions which works well for small momentum transfers deviates from experimental measurements by up to two orders of magnitude for $Q^2 > 6 fm^{-2}$. It is found that the explicit inclusion of two-body mechanisms, where the photon is absorbed on one nucleon and the pion is emitted from another nucleon can restore agreement with the data.

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Reactions on the trinucleon are an ideal testing ground to search of effects that go beyond the Impulse Approximation since realistic correlated three-body wave functions are available that are reliable even at high nuclear momentum transfers. Previous theoretical investigations [1–3] of pion scattering and pion photoproduction on the trinucleon systems were mostly based on the multiple-scattering approach carried out in momentum space. Using the impulse approximation (IA) and realistic nuclear wave functions the nonlocalities of the pion-nuclear interaction and exact treatment of Fermi motion have been taken into account. Within a coupled-channels framework it has become possible to consistently describe π^+ and coherent π^0 photoproduction as well as elastic and charge-exchange pion scattering on ^{3}He and ^{3}H and obtain a good description of experimental measurements at momentum transfers of $Q^2 < 6fm^{-2}$. Thus, the calculations have reached a point where the conventional one-body aspects are treated on a rather accurate level.

However, at higher momentum transfers large discrepancies appear between measurements and theoretical calculations [1]. In the region of $Q^2 > 8fm^{-2}$, calculations dramatically fail to explain existing $^3He(\gamma,\pi^+)^3H$ data, underestimating them by up to two orders of magnitude. Similar discrepancies were found in coherent π^0 photoproduction on ^3He and ^3H [4], in elastic pion scattering at backward angles [2,5], as well as in pion single charge exchange and pion induced eta production on ^{3}He at higher energies [2,6].

In general, it has been known for a long time that imposing gauge invariance in electromagnetic reactions with the nucleus will generate two-body meson exchange currents via, e.g., the Siegert theorem. Since the bound nucleons are off-shell, the one-body currents are no longer conserved and, therefore, meson exchange currents are required in order to fulfill gauge invariance. Very recently, this method was demonstrated on the specific reaction of pion photoproduction on finite nuclei [7]. Using general requirements such as current conservation in the nuclear electromagnetic vertex ref. [7] obtained general expressions for two-body correction terms similar to meson exchange currents in electron scattering. However, it is also well-known that this procedure generates only the convection part of the two-body currents since the magnetic parts are transverse by definition and, therefore, fulfil

gauge invariance separately.

The goal of this work is to study genuine two-body mechanisms in pion photoproduction on ^{3}He which do not appear in a standard distorted wave impulse approximation framework (DWIA). Our main requirement is to derive such two-body operators in a way that is consistent with the usual one-body operator describing the process on the single nucleon. This consistency requirement can be satisfied straightforwardly by starting from an effective Lagrangian for the pion-nuclear production process and to introduce the electromagnetic field by minimal substitution. This method not only guarantees gauge invariance but also allows the explicit calculation of contributions from the magnetic moments.

Using the impulse approximation for the nuclear pion emission amplitude one can write

$$T_\pi = \frac{f}{m_\pi} \int d\vec{r}_\pi \phi^\dagger_\alpha(\vec{r}_\pi) < f \mid \sum_{j=1}^{A} \vec{\sigma} \cdot \overleftarrow{\nabla}_\pi \ \tau_\alpha(j)\, \delta(\vec{r}_\pi - \vec{r}_j) \mid i > , \tag{1}$$

where $f/m_\pi = g/2M$ with $g^2/4\pi = 14$, M denotes the nucleon mass, $\vec{\tau}$ is the nucleon isospin operator and $\phi_\alpha(\vec{r}) = \varphi_\alpha e^{i\vec{q}\cdot\vec{r}}$ is the pion wave function with the three isospin components $\alpha = 1, 2, 3$.

The main part of the charged pion photoproduction amplitude - seagull and pion exchange terms - can be obtained by minimal substitution $\vec{\nabla}_\pi \to \vec{\nabla}_\pi - ie\vec{A}$ (where $\vec{A} = \vec{\epsilon} e^{i\vec{k}\cdot\vec{r}}$ is the electromagnetic vector potential with polarization vector $\vec{\epsilon}$ and photon momentum $\vec{k}$) in the $\vec{\sigma} \cdot \overleftarrow{\nabla}_\pi$-operator and in the pion wave function. Since the corresponding procedure is well known and straightforward, we present here only the treatment of the more complicated nucleon pole (dispersive or two-step) terms. In our approach, we construct this part of the pion photoproduction operator by introducing the electromagnetic field in the nuclear wave function which satisfies the Schrödinger equation. Then the initial nuclear wave function can be written as

$$\mid i > \to \mid i > + \frac{1}{\mathcal{E}_i - \hat{H}_0} \hat{H}_{em} \mid i > , \quad \hat{H}_{em} = \frac{ie}{2M} \sum_{n=1}^{A} (\vec{\nabla}_n \cdot \vec{A} + \vec{A} \cdot \vec{\nabla}_n) \, , \tag{2}$$

where $\hat{H}_0 = \sum_{n=1}^{A} \hat{\vec{p}}_n^{\,2} / 2M$ is the Hamiltonian for noninteracting nucleons. Note that at this step in the derivation of Eq. (2) we retained only the electromagnetic convection current

which is crucial for gauge invariance and neglected minimal substitution in the nucleon-nucleon interaction.

In order to get a standard one-body operator used in the IA [8] we will apply the closure approximation in the evaluation of the nuclear propagator $1/(\mathcal{E}_i - \hat{H}_0)$ with the mean nuclear excitation energy $\bar{E}$:

$$\frac{1}{\mathcal{E}_i - \hat{H}_0} \to \frac{1}{\mathcal{E}_i - \bar{E}} = \frac{2M}{2ME_\gamma - \vec{k}^2} \, . \tag{3}$$

Thus, in the expansion of the nuclear propagator only the leading term was retained and the difference between the nuclear ground and excited states energies was neglected.

Substituting Eqs. (2,3) in Eq. (1) yields

$$T^{(s)}_{\pi\gamma} = \frac{-ief}{m_\pi} \varphi^\dagger_\alpha < f \mid \sum_{j=1}^{A} \hat{\tau}_\alpha(j) \vec{\sigma}_j \cdot \vec{q} \, e^{-i\vec{q}\cdot\vec{r}_j} \sum_{n=1}^{A} \hat{e}(n) \, e^{i\vec{k}\cdot\vec{r}_n} \frac{(2\hat{\vec{p}}_n + \vec{k}) \cdot \vec{\epsilon}}{2ME_\gamma - \vec{k}^2} \mid i > , \tag{4}$$

where the operator $\hat{\vec{p}}_n = -i\vec{\nabla}_n$ acts on the initial nuclear state $\mid i >$ and is associated with the initial nucleon momentum $\vec{p}_i$. The isospin operator for the nucleon charge is $\hat{e} = (1 + \tau_3)/2$.

The expression for the dispersive term in Eq. (4) contains matrix elements of one-body as well as two-body operators. The former corresponds to the case $n = j$ and is shown in Fig. 1a. This term is identical to the s-channel amplitude in pion photoproduction on a single nucleon [8]. The two-body part of Eq. (4) corresponds to the case $n \neq j$ and is shown in Fig. 1c. This corresponds to a new class of diagrams that do not appear in the IA.

Applying the same procedure described above for the final nuclear state $\mid f >$ we also obtain the pion photoproduction amplitude in the u-channel,

$$T^{(u)}_{\pi\gamma} = \frac{ief}{m_\pi} \varphi^\dagger_\alpha < f \mid \sum_{n=1}^{A} \hat{e}(n) \frac{(2\hat{\overleftarrow{p}}_n - \vec{k}) \cdot \vec{\epsilon}}{2ME_\pi + \vec{q}^2} e^{i\vec{k}\cdot\vec{r}_n} \sum_{j=1}^{A} \hat{\tau}_\alpha(j) \vec{\sigma}_j \cdot \vec{q} \, e^{-i\vec{q}\cdot\vec{r}_j} \mid i > , \tag{5}$$

where the operator $\hat{\overleftarrow{p}}_n = i \overleftarrow{\nabla}_n$ acts on the final nuclear state $\mid f >$ and is associated with the final nucleon momentum $\vec{p}_f$. Again, the one-body part of this amplitude (case $n = j$) describes the elementary process in the u-channel (Fig. 1b), while the two-body part (case $n \neq j$) is shown in Fig. 1d.

Finally, choosing the Coulomb gauge, $\vec{k}\cdot\vec{\epsilon}=0$, we obtain the standard expression for the one-body part of the dispersive amplitude which is traditionally used in the IA. Furthermore, we generate a novel two-body mechanism, that is given by

$$T^{conv.}_{\pi\gamma}(2) = T^{(s)}_{\pi\gamma}(2) + T^{(u)}_{\pi\gamma}(2) = \frac{-ief}{m_\pi}\varphi^\dagger_\alpha < f \mid \sum_{j\neq n}^{A} \hat{e}(n)\hat{\tau}_\alpha(j)\,\vec{\sigma}_j\cdot\vec{q}\,e^{i\vec{k}\cdot\vec{r}_n - i\vec{q}\cdot\vec{r}_j}$$
$$\times\left(\frac{2\hat{\vec{p}}_n\cdot\vec{\epsilon}}{2ME_\gamma - \vec{k}^2} - \frac{2\overleftarrow{\hat{p}}_n\cdot\vec{\epsilon}}{2ME_\pi + \vec{q}^2}\right) \mid i > \ . \tag{6}$$

The contributions from the magnetic interaction due to the magnetic moments of the proton, $\mu_p = 2.79$, and the neutron, $\mu_n = -1.91$, can be obtained analogously. The corresponding Hamiltonian is

$$\hat{H}_{magn.} = \frac{e}{2M}\sum_{n=1}^{A}\hat{\mu}(n)\,\vec{\sigma}_n\cdot\vec{B}(\vec{r})\,, \tag{7}$$

where $\vec{B}(\vec{r}) = \vec{\nabla}\times\vec{A} = i[\vec{k}\times\vec{\epsilon}]\,e^{i\vec{k}\cdot\vec{r}}$. In analogy to the convection part considered above the two-body mechanism due to the nucleon magnetic moments reads

$$T^{(magn.)}_{\pi\gamma}(2) = \frac{ef}{m_\pi}\varphi^\dagger_\alpha < f \mid \sum_{j\neq n}^{A}\hat{\tau}_\alpha(j)\hat{\mu}(n)\,\vec{\sigma}_j\cdot\vec{q}\,\vec{\sigma}_n\cdot[\vec{k}\times\vec{\epsilon}]$$
$$\times\left(\frac{1}{2ME_\gamma - \vec{k}^2} - \frac{1}{2ME_\pi + \vec{q}^2}\right)e^{i\vec{k}\cdot\vec{r}_n - i\vec{q}\cdot\vec{r}_j} \mid i >\,, \tag{8}$$

where the magnetic isospin operator is defined as $\hat{\mu} = \mu_p(1+\tau_3)/2 + \mu_n(1-\tau_3)/2$.

Even at large momentum transfer the energy difference $E_\gamma - E_\pi$ is only a few MeV, therefore, to leading order the s- and u-channel propagators cancel each other. This has also been found by Levchuk and Shebeko [9] in a nuclear pion photoproduction study that investigated effects beyond the impulse approximation. However, as we demonstrate below, it is exactly the difference between the s- and u-channel contribution of order $(\vec{k}^2+\vec{q}^2)/(2ME_\gamma)$ that is very important in the high momentum transfer region.

We begin our discussion by considering pion rescattering effects. Single rescattering, shown in Fig. 1e,f, is known to be incomplete especially in the Δ resonance region, where the pion-nucleon interaction is very strong. Within a multiple scattering framework [10] we have studied pion scattering and photoproduction on ^{3}He in detail in Refs. [1,2]. In

Fig. 2 we show results of our previous work, comparing a PWIA calculation without any pion rescattering with a DWIA computation with full pion-nucleus final state interaction including single charge exchange. The latter one describes the data well up to $Q^2 \approx 6 fm^{-2}$. Since the standard multiple scattering framework contains contributions from the trinucleon ground state only, we have estimated the additional contributions from the coupling to the break-up channels using closure approximation. However, comparing the dashed and dash-dotted curves in Fig. 2, it is clear that the disagreement at high momentum transfer can not be improved significantly by contributions from pion rescattering alone.

This situation changes dramatically, once the novel two-body mechanisms, shown in Fig. 1c and d, are taken into account. As shown in Fig. 2, including these two-body amplitudes of Eqs. (6) and (8) within the multiple scattering framework raises the cross section by up to two orders of magnitudes, thus, these two-body mechanisms in fact become dominant for $Q^2 > 7 fm^{-2}$. This effect removes most of the discrepancy between theory and experiment. Our analysis indicates that this large enhancement comes mainly from the isovector magnetic interaction of the two-body operator defined by Eq. (8). As mentioned above, even as the leading terms of the s- and u-channel propagators cancel each other, the next higher-order term, proportional to $(\vec{k}^2 + \vec{q}^2)/(2ME_\gamma)$, becomes significant in the high momentum transfer region. Note, that this cancellation of the leading two-body terms was also found by Jennings [13] in a study of pion-deuteron scattering. On the other hand, the two-body contribution arising from the convection part of the electromagnetic current (see Eq. (6)) is significantly smaller because it does not have the enhancing factor of the isovector moment and, moreover, is of nonlocal nature.

In Fig. 3 we illustrate the importance of the two-body mechanisms for the differential cross sections at backward angles. Starting at a photon energy of $E_\gamma = 300$ MeV, the contributions of diagrams Fig. 1c,d become visible. At $E_\gamma = 500$ MeV and $\theta_\pi > 150^0$, corresponding to $Q^2 > 16 fm^{-2}$, the two-body mechanisms increase the differential cross section by up to two orders of magnitude.

Finally, we demonstrate in a simple qualitative approach how momentum sharing among

the nucleons inside the nucleus enhances the cross section in the high momentum transfer region. In the simple harmonic oscillator model the contribution from the one-body operator is determined by the well-known gauss form factor

$$F^{(1)}(\vec{Q}) = \int d\vec{p}\, d\vec{\mathcal{P}} \Psi_S^+(\vec{p}, \vec{\mathcal{P}} - \tfrac{2}{3}\vec{Q})\, \Psi_S(\vec{p}, \vec{\mathcal{P}}) = e^{-b^2 Q^2/6}\,, \tag{9}$$

where $\vec{Q} = \vec{k} - \vec{q}$ is the nuclear momentum transfer which in the IA has to be absorbed by a single nucleon and Ψ_S is the S-shell wave function defined as $\Psi_S(\vec{p}, \vec{\mathcal{P}}) = N exp(-b^2(p^2 + \frac{3}{4}\mathcal{P}^2))$. For the matrix element that contains the two-body operator we arrive at the expression

$$F^{(2)}(\vec{\mathcal{K}}, \vec{Q}) = \int d\vec{p}\, d\vec{\mathcal{P}} \Psi_S^+(\vec{p} + \tfrac{1}{2}\vec{\mathcal{K}}, \vec{\mathcal{P}} + \tfrac{1}{3}\vec{Q})\, \Psi_S(\vec{p}, \vec{\mathcal{P}}) = e^{-b^2 Q^2/24}\, e^{-b^2 \mathcal{K}^2/8} \tag{10}$$

with $\vec{\mathcal{K}} = \vec{k} + \vec{q}$. Therefore, sharing of momentum transfer among two nucleons leads to a much smaller exponential argument compared to the one-body case. This becomes obvious at high energies and backward angles where the photon and pion momenta have similar magnitude but opposite sign, leading to $\vec{\mathcal{K}} \approx 0$. For example, in the case of pion photoproduction at E_γ=400 MeV, $\theta_\pi = 137$ and 180 we have $F^{(2)}/F^{(1)} \approx 340$ and 3000 respectively.

In conclusion, we have studied the dramatic disagreement between high-Q (γ, π^+) data on ^3He and DWIA calculations that underpredict these data by up to two orders of magnitude. We have resolved this long-standing discrepancy by including explicit two-body mechanisms that go beyond the Impulse Approximation. These two-body terms, where the photon is absorbed on one nucleon and the pion is emitted from another, allow momentum transfer sharing between the two nucleons and dominate the cross section for $Q^2 > 7fm^{-2}$. The most important contribution of these two-body currents are found to be due to the magnetic interaction. Pion rescattering, which is important in the kinematic region of $Q^2 < 6fm^{-2}$, cannot account for the observed discrepancy, even if the coupling to the break-up channels is included in the intermediate states.

From our derivation it is clear that these novel two-body mechanisms are not a special feature of the $^3He(\gamma, \pi^+)^3H$ process but should play an important role in all scattering and

production processes at high momentum transfer, as long as the nucleus remains in its ground state. Such processes are coherent π^0 photoproduction on the deuteron or ^4He, (γ, η) on light systems, nuclear Compton scattering and, furthermore, meson elastic scattering and single charge exchange processes. The new high energy, 100% duty factor electron accelerators MAMI at Mainz, ELSA at Bonn and CEBAF are the ideal tools to investigate this field of two-body effects and nucleon-nucleon correlations.

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FIGURES

FIG. 1. Diagrams for the dispersive and pion rescattering terms in nuclear pion photoproduction.

FIG. 2. Differential cross section at $\Theta_{c.m.} = 137^0$ as a function of nuclear momentum transfer Q^2. The dotted (dashed) curves show the PWIA (DWIA) results obtained with Faddeev wave functions [12]. The dash-dotted curve includes the corrections due to the coupling with the break-up channels and the full line shows our complete calculation with the additional novel two-body contribution of Fig. 1c,d. The experimental data are from Ref. [11]

FIG. 3. Pion angular distribution at $E_\gamma = 300$, 400 and 500 MeV. The notations of the curves are the same as in Fig. 2. The experimental data are from Refs. [11,14].